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Application of Flexible Functional Forms to Substitutability among Metals in U.S. Industries

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A new functional form — the Symmetric Generalized Mcfadden Cost Function (SGM) — is used to estimate substitutability among metals in five U.S. industries.

This paper — a product of the International Commodity Markets Division, International Economics Department — is part of a larger effort in PRE to investigate the decline since the mid-1970s in the intensity of use of metals in the industrial countries. Whether or not this change in industrial countries' demand for metals is permanent is of great importance for the developing country producers of the raw materials. Copies of this paper are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Sarah Lipscomb, room S7-062, extension 33718 (41 pages with figures and tables).

Qian reports on the use of a new functional form — the Symmetric Generalized Mcfadden Cost Function (SGM) — to estimate substitutability among metals in five U.S. industries. The SGM specification has the advantage of imposing curvature conditions globally on the cost function, thus ensuring that the results satisfy basic, widely believed economic theory.

For the first time, this study assumes separability in estimating an SGM system, and experiments with a "bootstrapping" technique to estimate the standard errors of parameters derived from flexible functional forms. In many cases, the SGM results are comparable to results from an earlier study that used the translog functional form to estimate the demand elasticities of metals with regard to price.

The procedure is also able to distinguish between direct elasticities for particular metals and overall elasticities for metals as a group under the separability assumption adopted for the metals subgroup.

Qian provides empirical evidence of structural change in U.S. industry. A jump in the own-price elasticities of energy during the sample period coincided with a sharp increase in oil prices.

The SGM flexible functional form found aluminum and steel to be complementary in four out of five industries but suggests that they are substitutes in the technically-compensated sense: when total metals use is constant, an increase in the price of one metal reduces consumption of that metal and increases consumption of the other.

Use of the bootstrapping technique provided insights into the stability of the elasticity estimates. The results are promising at the aggregate level when the number of free parameters is not large compared to the sample size. Bootstrapping also clarifies the problem at the disaggregated level where most elasticities are not significantly different from zero.

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by
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Application of Flexible Functional Forms to Substitutability among Metals in U.S. Industries

Ying Qian[1]

I. Introduction

This paper applies the recently developed theory of flexible functional forms to the estimation of the substitutability among metals in five major U.S. industries. Priovolos and Dunietz (1987) used the translog cost function approach to estimate substitution elasticities between metal raw materials in U.S. manufacturing industries, and found that the theoretical curvature condition, i.e., the concavity condition in the cost function, was not satisfied in many cases. The method introduced here is able to impose curvature conditions globally in the framework of cost function estimation, thus ensuring that the results fulfill the basic and widely believed economic theory.

Specifically, this paper presents results derived using the Symmetric Generalized McFadden Cost Function[2] (SGM) and the concept of separability in the estimation of elasticities of substitution between metals in several U.S. industries, and compare results against outcomes derived using the translog functional form. The data set developed by Priovolos and Dunietz is used.

The paper investigates the constancy of the various elasticity estimators by calculating and plotting their confidence intervals[3] over the sample period. These results can provide evidence for the debate over whether structural changes have been experienced in recent years. Unfortunately, there is no guarantee that the elasticity estimates from the flexible functional form will be stable, because either the reality is subject to structural changes or

[1]I am grateful to R. Duncan, T. Priovolos and D. Mitchell for their valuable comments and editing, to T. Dunietz for her help on my understanding of their earlier study, to T. Sihsobhon for his computer support, and participants in the IECCM seminar; finally, in particular, to R. Kopp at Resources For the Future who introduced me into this area.

[2]See Diewert and Wales (1987).

[3]Asymptotic or Bootstrap standard errors for elasticities. See Green and Hahn (1987).

the limited degrees of freedom due to introducing too many free parameters in the system could diminish the efficiency of the estimators. For this latter reason, some researchers have argued that flexible forms do not always generate empirically credible elasticity estimates.

Part II surveys and summarizes recent literature on flexible functional forms. Part III utilizes the weak separability assumption in the derivation of input factor demand functions of the two-stage budgeting process and their related input price elasticities. Part IV provides a description of the data and specification of the model to be estimated. Part V presents empirical results in the form of cross price elasticities. In part VI, the Bootstrapping technique and its application are discussed. Part VII presents the conclusion.

II. The Symmetric Generalized McFaoden Cost Function

Suppose the production function for a firm or an industry in period t is $y=f^{t*}(x_1,x_2,\ldots,x_n)$, where y is the optimal output and $X=(x_1,\ldots,x_n)^T$ is the vector of inputs. Given $P=(p_1,\ldots,p_n)^T \gg 0_N$ as the input price vector, the cost function c^* will be defined as:

(1) $$c^*(P,y,t)=\min_X\{P^TX: f^{t*}(X)\geq y,\ X\geq 0_N\}$$

As conventional wisdom shows, c^* should be a linear homogeneous and concave function in P. Denote $\nabla_P c^*$ as a column vector of the first order partial derivative of c with respect to P, and $\nabla_{PP}^2 c^*$ as an N by N matrix of the second order partial derivative of c^* with respect to P. Adopting the twice continuously-differentiable assumption about c^*, and Young's theorem, the following has to hold:

(2) $$\nabla_{PP}^2 c^*(P^*,y^*,t^*)=[\nabla_{PP}^2 c^*(P^*,y^*,t^*)]^T$$

The concavity in prices implies $\nabla_{PP}^2 c^*(P^*,y^*,t^*)$ is a negative semi-definite matrix.

In order to avoid the implicit restrictions imposed by using certain functional forms, i.e., linear or CES, economists have developed various types of flexible functional forms. Flexibility is defined as having enough free parameters to be able to approximate an arbitrary, twice-continuously-differentiable cost function to the second order. There is no limitation on the demand elasticity yielded by such cost functional forms.

Denote c as the estimated cost function. c is flexible, iff it has enough free parameters to satisfy:

(3) $$c(P^*,y^*,t^*)=c^*(P^*,y^*,t^*)$$

(4) $$\nabla c(P^*,y^*,t^*)=\nabla c^*(P^*,y^*,t^*)$$

(5) $$\nabla^2 c(P^*,y^*,t^*)=\nabla^2 c^*(P^*,y^*,t^*)$$

Where ∇c and $\nabla^2 c$ are first and second partial derivatives with respect to all variables inside the parentheses.

The most widely used flexible functional forms to date are the Translog and Generalized Leontief functions.

The Translog formulation is as follows:

(6) $$\ln c(P,y,t)=a_0+\Sigma^N_{i=1}a_i\ln p_i+a_y\ln y+a_t t +\tfrac{1}{2}\Sigma^N_{i=1}\Sigma^N_{j=1}a_{ij}\ln p_i\ln p_j+\Sigma^N_{i=1}a_{iy}\ln p_i\ln y+\Sigma^N_{i=1}a_{it}t\ln p_i$$

$$+\tfrac{1}{2}a_{yy}\ln y \ln y + a_{yt}t\ln y + \tfrac{1}{2}a_{tt}t^2$$

$$a_{ij}=a_{ji} \text{ for all } i,j.$$

Linear homogeneity in input price P will be insured if the following restrictions on parameters hold:

(7) $$\Sigma^N_{i=1}a_i=1;\ \ \Sigma^N_{j=1}a_{ij}=0,\ i=1,\ldots,N;\ \ \Sigma^N_{i=1}a_{iy}=0;\ \ \Sigma^N_{i=1}a_{it}=0$$

If the following conditions also hold, the associated production function would exhibit constant returns to scale:

(8) $$a_y=1;\ \ a_{iy}=0,\ i=1,2,\ldots,N\text{-}1;\ \ a_{yy}=0;\ \ a_{yt}=0$$

Translog cost functions often fail to satisfy the property of concavity in prices. Researchers have tried to impose the concavity restraint,[4] but doing this is not as convenient as incorporation in other flexible forms. However, the translog functions do have a very useful property: the differentiation of (6) with respect to $\ln p_i$ gives the linear share equation for input i; thus it is easy to carry out the estimation process.

The Generalized Leontief Cost Function (GLC) is represented as follows:

(9) $$c(P,y,t)=\Sigma^N_{i=1}\Sigma^N_{j=1}b_{ij}p_i^{1/2}p_j^{1/2}y+\Sigma^N_{i=1}b_ip_i+\Sigma^N_{i=1}b_{it}p_ity$$
$$+b_t(\Sigma^N_{i=1}\alpha_ip_i)t+b_{yy}(\Sigma^N_{i=1}\beta_ip_i)y^2+b_{tt}(\Sigma^N_{i=1}\gamma_ip_i)t^2y$$

$$b_{ij}=b_{ji} \text{ for } i,j=1,2,\ldots,N.$$

GLC is flexible, and unlike the Translog, it is easy to impose the constraint of concavity in input prices. That is, all the b_{ij} $(i\neq j)$ in (9) are negative. However, this functional form is unduly restrictive because complementarity is not possible between any pair of inputs.

The Generalized McFadden Cost Function (GMC) is defined as follows:

(10) $$c^1(P,y,t)=g^1(P)y+\Sigma^N_{i=1}b_{ii}p_iy+\Sigma^N_{i=1}b_ip_i+\Sigma^N_{i=1}b_{it}p_ity$$
$$+b_t(\Sigma^N_{i=1}\alpha_ip_i)t+b_{yy}(\Sigma^N_{i=1}\beta_ip_i)y^2+b_{tt}(\Sigma^N_{i=1}\gamma_ip_i)t^2y$$

Function g^1 is defined as:

$$g^1(P)=(\tfrac{1}{2})p_1^{-1}\Sigma^N_{i=2}\Sigma^N_{j=2}c_{ij}p_ip_j \quad \text{where } c_{ij}=c_{ji} \text{ for } i,j=2,\ldots,N$$

Notice that GMC and GLC are identical, except that the term $\Sigma^N_{i=1}\Sigma^N_{j=1}b_{ij}p_i^{1/2}p_j^{1/2}y$ in (9) has been replaced by $g^1(P)y$ and $\Sigma^N_{i=1}b_{ii}p_iy$ in (10). It is easy to see that $\nabla_{PP}^2c^1(P,y,t)=\nabla_{PP}^2g^1(p)y$. Input 1 plays an asymmetric role in the definition of g^1, it is used to ensure the property of homogeneity in the price vector in (10).

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[4]See Jorgenson and Fraumeni (1981).

GMC is equivalent to GLC in terms of its flexibility, ease of estimation and hypothesis testing capabilities. The noteworthy feature of GMC is that the concavity restriction can be imposed globally.[5] But because input 1 can be replaced by input k at the researcher's disposition, choosing different inputs to play an asymmetric role may yield different or conflicting results from empirical estimation although in theory it should not matter.

The Symmetric Generalized Mcfadden Cost Function (SGM) is defined as:

$$(11)\qquad c(P,y,t)=g(P)y+\Sigma^N_{i=1}b_{ii}p_iy+\Sigma^N_{i=1}b_ip_i+\Sigma^N_{i=1}b_{it}p_ity$$
$$+b_t(\Sigma^N_{i=1}\alpha_ip_i)t+b_{yy}(\Sigma^N_{i=1}\beta_ip_i)y^2+b_{tt}(\Sigma^N_{i=1}\tau_ip_i)t^2y$$

Function g is defined as:

$$g(P)=(1/2)P^TSP/\theta^TP$$

where $S=S^T$ is an N by N symmetric, negative semi-definite matrix and $\theta=[\theta_1,\ldots,\theta_n]^T>0_N$ is a vector of pre-selected non-negative constants, not all equal to zero. Thus, g(P) is globally concave over the positive orthant as well as c(P,y,t).

The functional form c has the advantage of being symmetric in its handling of inputs if θ is chosen to treat all inputs symmetrically. However, it has N more unknown parameters than GMC. N more restrictions on the elements of the S matrix are needed in order to equally identify all the parameters in this SGM setting.

Suppose some price can be chosen as $P^*\gg 0_N$; then the extra N restrictions that can be placed on the elements of the S matrix are:

$$(12)\qquad SP^*=0_N$$

Diewert and Wales(1987) proved, that if $\theta^TP^*>0$, $\alpha^TP^*\neq 0$, $\beta^TP^*\neq 0$ and $\tau^TP^*\neq 0$ then c defined by (11) is a flexible cost function at the point P^* which satisfies (12). If S is not negative semi-definite, there is a way to impose it[6] by setting $S=-AA^T$ without eliminating the flexibility of c. A^T is an upper triangular matrix where $A^TP^*=0_N$.

Applying Shepherd's lemma to the cost function gives the derived input demand $x_i=\partial c/\partial p_i$. In this SGM formulation, x_i, in the nonlinear form, is given as:

$$(13)\qquad x_i=(\Sigma^N_{j=1}s_{ij}p_jy)/(\Sigma^N_{k=1}\theta_kp_k)-(1/2)\theta_i((\Sigma^N_{k=1}\Sigma^N_{j=1}p_kp_js_{kj})y/(\Sigma^N_{k=1}\theta_kp_k)^2)$$

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[5]See Diewert and Wales (1987) for details.

[6]See Wiley, Schmidt and Bramble (1973).

$$+b_{ii}y+b_i+b_{it}ty+b_t\alpha_i t+b_{yy}\beta_i y^2+b_{tt}\tau_i t^2 y$$

for $i=1,\dots,N$

where the unknown parameters are s_{ij}, b_{ii}, b_i, b_{it}, b_t, b_{yy} and b_{tt}. θ_k, α_i, β_i and τ_i are pre-selected by the researcher.

If there are no restrictions on these pre-selected and unknown parameters, the SGM does not impose constant returns to scale in the dual production function. The hypothesis is testable using the estimates of the parameters in equation (13).

III. Separability and Elasticities

As Part II indicates, the flexible cost function requires a large number of free parameters in its estimation. If the time series data is short, degrees of freedom will impose an upper limit on the number of inputs the system can handle.

Nevertheless, if separability is assumed, the sample size may not be so restrictive. Separability implies that the total optimization is equivalent to a two-stage optimization process, where the optimal mix of inputs in the detailed level is chosen in the first stage, and in the second stage the optimal amount of inputs at the aggregate level is determined.[7] This assumption is extremely useful because the substitution possibilities within a particular group of inputs can be addressed separately and without considering the substitution between one of the inputs in the group and inputs in other groups, thus greatly reducing the number of required free parameters. Historically, separability has played an important role in the specification of functional forms; for example, the Cobb-Douglas and CES functions are explicitly strongly separable.

To define separability in a mathematical context, first denote the set of n inputs by $N=\{1,...,n\}$. A partition S of N is given by $\{N_1,...,N_S\}$ where $N=N_1 \cup N_2 \cup ... \cup N_S$, and $N_r \cap N_t$ is empty for $r \neq t$. If the marginal rate of substitution between a pair of inputs is independent from changes in the level of another input, i.e.,

$$\partial(f_i/f_j)/\partial v_k=0$$

then f is separable. The function f is strongly separable with respect to the partition S if the above form holds for all $i \in N_r$, $j \in N_t$, and $k \notin N_r \cup N_t$. The function is weakly separable with respect to the partition S if the above holds for all $i,j \in N_r$, and $k \notin N_r$.

The weak separability assumption is applied in our SGM estimation of input substitution in U.S. industrial sectors. The partition at the aggregate level of inputs is in terms of energy, capital, labor and metals. There are seven basic inputs in the metals group: aluminum, copper, nickel, steel, tin, zinc and lead.

The separability assumption is plausible for metals. They are all used as material

[7] See Magnus and Woodland (1984).

in the production process, and have distinct features compared to other types of input (e.g., labor or capital). For this reason, relative prices among metals have been playing the dominant role in determining the shares of the various metals. However, in extreme cases and for some specific pairs of metals, the weak separability assumption does break down. It appears that one example is the impact of the oil shock on the relationship between aluminum and steel. The sudden oil price increase reduced energy consumption. At the same time, auto-makers may have used more aluminum parts to substitute for parts made of steel. Thus, the marginal rate of substitution between aluminum and steel may have been changed because of the oil shock and the separability assumption violated.

The separability assumption is testable in the flexible functional form system [illegible]ng as the time series is long enough to allow unrestricted regression, i.e., without utilization of the weak separability assumption. This test could not be carried out in this paper because the sample size is too small.

Rewriting the SGM cost function (11) as an unit cost function:

$$(11)' \quad c(P,y,t)/y = g(P) + \Sigma^N_{i=1} b_{ii} p_i + \Sigma^N_{i=1} b_i p_i / y + \Sigma^N_{i=1} b_{it} p_i t + b_t(\Sigma^N_{i=1} \alpha_i p_i) t/y + b_{yy}(\Sigma^N_{i=1} \beta_i p_i) y + b_{tt}(\Sigma^N_{i=1} \tau_i p_i) t^2$$

The corresponding input demand function re-scaled by output y is:

$$(13)' \quad z_i = (\Sigma^N_{j=1} s_{ij} p_j)/(\Sigma^N_{k=1} \theta_k p_k) - (1/2)\theta_i((\Sigma^N_{k=1} \Sigma^N_{j=1} p_k p_j s_{kj})/(\Sigma^N_{k=1} \theta_k p_k)^2) + b_{ii} + b_i/y + b_{it} t + b_t \alpha_i t/y + b_{yy} \beta_i y + b_{tt} \tau_i t^2$$

for $i = 1,\ldots,N$

where $z_i = x_i y^{-1}$.

Let p_m be the price index of a sub-group of inputs.[8] p_m can also be considered as the unit cost of the aggregate measure of the sub-group of inputs: x_m. Using the same SGM formulation for p_m:

$$(14) \quad p_m(R,x_m,t) = g_m(R) + \Sigma^V_{i=1} b_{mii} r_i + \Sigma^V_{i=1} b_{mi} r_i / x_m + \Sigma^V_{i=1} b_{mit} r_i t + b_{mt}(\Sigma^V_{i=1} \alpha_{mi} r_i) t/x_m + b_{myy}(\Sigma^V_{i=1} \beta_{mi} r_i) x_m + b_{mtt}(\Sigma^V_{i=1} \tau_{mi} r_i) t^2$$

where $g_m(R) = (1/2) R^T S_m R / \theta_m{}^T R$ and $S_m = -A_m A_m{}^T$. x_m is a function of $(x_{mi},\ldots,x_{mv})$: the vector of sub-group demand. In order to preserve the comparability with (11), all parameters in (14) and (11) have the same name, except the subscript m. $R = (r_1,\ldots\ldots,r_v)^T \gg 0_V$ is the price

[8]In this paper, it refers to metals.

vector in this sub-group, with dimension V. The V more restrictions on the p_m formulation is $S_m R^* = 0_V$.

Using the chain rule:[9]

$$\partial c/\partial r_i = (\partial c/\partial p_m)(\partial p_m/\partial r_i)$$

$$x_{mi} = x_m(\partial p_m/\partial r_i)$$

So the input demand for x_{mi} re-scaled by x_m is:

(15) $$z_{mi} = (\Sigma^V_{j=1} s_{mij} r_j)/(\Sigma^V_{k=1}\theta_{mk} r_k) - (1/2)\theta_{mi}((\Sigma^V_{k=1}\Sigma^V_{j=1} r_k r_j s_{mkj})/(\Sigma^V_{k=1}\theta_{mk} r_k)^2) + b_{mii} + b_{mi}/x_m + b_{mit}t + b_{mt}\alpha_{mi}t/x_m + b_{myy}\beta_{mi}x_m + b_{mtt}\tau_{mi}t^2$$

for $i = 1,\dots,V$

where $z_{mi} = x_{mi}x_m^{-1}$. Notice (15) and (13)' have identical formulations, and the separability provides the ease of estimating the SGM cost function form in the context of two-level optimization.

The second derivative of (11) with respect to prices on the first or aggregate level of input $(x_1,\dots\dots,x_n)$ is calculated as follows:

(16) $$\nabla_{PP}^2(c(P,y,t)/y) = \nabla_{PP}^2 g(P) = S/\theta^T P - SP\theta^T/(\theta^T P)^2 - \theta P^T S/(\theta^T P)^2 + (\theta P^T)S(P\theta^T)/(\theta^T P)^3$$

On the second, or detailed, level $(x_{mi},\dots,x_{mv})$, the elasticity of x_{mi} with respect to r_j, $i,j = 1,\dots,V$, is defined as $(\partial x_{mi}/\partial r_j)(r_j/x_{mi})$, and to calculate $(\partial x_{mi}/\partial r_j)$ would involve the calculation of the following:

(17) $$\begin{aligned}\partial^2 c(P,y,t)/\partial R^2 &= \partial((\partial c/\partial p_m)(\partial p_m/\partial R)^T)/\partial R \\ &= (\partial(\partial c/\partial p_m)/\partial R)(\partial p_m/\partial R)^T) + (\partial(\partial p_m/\partial R)^T)/\partial R)(\partial c/\partial p_m) \\ &= (\partial^2 c/\partial p_m^2)(\partial p_m/\partial R)(\partial p_m/\partial R)^T + \nabla_{RR}^2 g_m(R)x_m\end{aligned}$$

where $\partial^2 c/\partial p_m^2$ is the diagonal element with respect to p_m in the matrix $\nabla_{PP}^2 g(P)y$. $(\partial p_m/\partial R)^T = (x_{m1}/x_m,\dots,x_{mr}/x_m)$. $\nabla_{RR}^2 g_m(R)$ would have the same expression as (16), except it is on $g_m(R)$ with respect to R. Notice that $\nabla_{RR}^2 g_m(R)$ reveals the substitution information directly on the detailed level, and the overall substitutability is acquired after the direct substitutability has been adjusted by the first term in (17).

To make it clearer, we can derive the formula of the cross elasticity of i^{th} metal demand x_{mi} with respect to j^{th} metal price r_j based on (17) as follows:

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[9]See Duncan and Binswager (1976).

$$(18) \qquad e_{mij} = (\partial z_m/\partial p_m)(r_j/z_m)z_{mj}+(\partial z_{mi}/\partial r_j)(r_j/z_{mi})$$

where $z_m = x_m/y$. The second term is similar to the elasticity calculation on the aggregate level, which can be considered as a direct cross elasticity within the metal sub-group. The overall cross elasticity of metal demand is obtained by adding the first item in (18), which is the own-price elasticity of aggregate metals with respect to r_j adjusted by the ratio of x_{mj} to x_m.

Testing this two-level flexible demand system cannot be completed if there are no associated standard error and confidence interval measures for the elasticity estimates. Unfortunately, the analytical expressions for the standard error of elasticity estimates may be difficult, if not impossible, to obtain. Thus, the asymptotic (Taylor series) approximation has been widely used. But, it may still be burdensome to derive the first order Taylor series of the expression for the standard errors of elasticity estimates in large systems such as estimated here. Moreover, the accuracy of asymptotic measures is questionable in small samples. It has been found[10] that the conventional asymptotic formula for estimating standard errors is sometimes too optimistic by a factor of nearly three when applied to a particular finite-sample problem.

Efron's bootstrap[11] estimates will be introduced in this paper to derive the standard errors for statistically derived quantities by means of Monte Carlo simulation. The model and the parameter values are set at those estimated for the real observations at hand, and the error distribution is taken to be the empirical distributions of the model residuals. The Bootstrap technique (bootstrap trial) is a process which generates a set of artificial data[12] from the specification of the fitted model and submits these data to the estimation procedure, which in turn yields a set of simulated parameters of interest. By repeating the bootstrap trial a number of times, we obtain a simulated distribution of parameters. As Freedman and Peters pointed out, bootstrap estimates have problems of their own; the calculation of the standard error still tends to be optimistic because the estimated residual tends to be smaller than the real disturbance term due to the effect of fitting. For example,

[10]See Freedman and Peters (1984).

[11]See Efron (1979).

[12]A series of observations on endogenous variables.

in some specifications the residuals can be scaled up by $[n/(n-p)]^{1/2}$, where n and p are the number of observations and parameters, respectively.

IV. Data and Specification

The data set used in this study was developed by Priovolos and Dunietz(1987). Their study selected five U.S. industrial sectors: Chemicals (SIC 28), Fabricated Metals, Cans and Containers (SIC 34), Machinery (SIC 35), Electricity (SIC 36) and Transportation (SIC 37). Four inputs were included in the cost functions of these industrial sectors: labor, capital, energy and metals. Metals were further decomposed into aluminum, copper, nickel, steel, tin, zinc and lead. These five industrial sectors account for 83% to 95% of total U.S. industrial demand for major metals: aluminum (83%), copper (95%), nickel (84%), steel (86%), tin (95%), zinc (86%) and lead (87%).

Major sources for this data set are: U.S. Department of Commerce, "Annual Survey of Manufactures: Statistics for Industry Groups and Industries, Fuels and Electric Energy Consumed" and U.S. Bureau of Mines, "Mineral Facts and Problems". Most of the metal prices are producer prices published by the U.S. Bureau of Mines. The steel price is a weighted index of steel bars, shapes, plates, wire, rails, black pipe, hot and cold rolled sheets and strip. The tin price is the New York market price for the 1963-75 period and the composite price thereafter. Lastly, the zinc price is the Western price up to 1981 and the U.S. high grade price thereafter.

Input expenditures for energy, labor and various metals in current terms are deflated by the appropriate U.S. wholesale price index which is equal to 1 in 1982. All the nominal prices of these inputs are deflated by the wholesale price index and normalized to be 1 at 1982. Input quantities for these inputs are obtained through the ratio of real expenditures to real normalized prices.

Total real expenditure or quantity of metals is the sum of individual real metal expenditures or quantities. The real normalized price of metals is the expenditure divided by the quantity.

The real price for capital is the ratio of real value added for capital to real capital stock. The value added for capital is the contribution of capital to value added, which equals the value added less payroll costs. The capital stock data in nominal terms were estimated by subtracting payroll expenditure from the value added. Estimates of capital stock in real terms were taken from Wharton Econometrics Forecasting Associates(WEFA). Like other prices of inputs, the price of capital is normalized to be 1 in 1982.

The estimated equations in each period are as follows:

(19) $$z_i=(\Sigma^N_{j=1}s_{ij}p_j)/(\Sigma^N_{k=1}\theta_k p_k)-(1/2)\theta_i((\Sigma^N_{k=1}\Sigma^N_{j=1}p_k p_j s_{kj})/(\Sigma^N_{k=1}\theta_k p_k)^2)$$
$$+b_{ii}+b_i/y+b_{it}t+\alpha_i t/y+\beta_i y+\tau_i t^2+u_i$$
$$\text{for } i=1,\dots,N$$

where $z_i=x_i y^{-1}$.

(20) $$z_{mi}=(\Sigma^V_{j=1}s_{mij}r_j)/(\Sigma^V_{k=1}\theta_{mk}r_k)-(1/2)\theta_{mi}((\Sigma^V_{k=1}\Sigma^V_{j=1}r_k r_j s_{mkj})/(\Sigma^V_{k=1}\theta_{mk}r_k)^2)$$
$$+b_{mii}+b_{mi}/x_m+b_{mit}t+\alpha_{mi}t/x_m+\beta_{mi}x_m+\tau_{mi}t^2+u_{mi}$$
$$\text{for } i=1,\dots,V$$

where $z_{mi}=x_{mi}x_m^{-1}$.

Instead of selecting α_i, β_i and τ_i exogenously in (13)', they were estimated in the model and to make the system more flexible, b_t, b_{yy} and b_{tt} were set to unity. The same treatment was also applied to (15).

Denoting $u=(u_1,\dots,u_N)^T$ to be the vector of the error terms, u is assumed to have a multivariate normal distribution with $E(u)=0$, $E(uu^T)=\Omega$, and the Ω is constant over time. The same assumption was applied to $u_m=(u_{m1},\dots u_{mv})$. Each θ_k and θ_{mk} was set equal to the sample mean of the corresponding inputs.

Equations (19) and (20) were estimated by the simultaneous, nonlinear, full information maximum likelihood method.

The computer package used in the estimation is GQOPT[13] which has been developed by Dr. Richard E. Quandt of Princeton University.

[13] A general purpose numerical optimization package in FORTRAN subroutines. The application program designed for estimation of the SGM functional form is available from the author upon request.

V. Empirical Results

Table 1 shows the expenditure shares of input factors at both the aggregated and disaggregated levels. On average, the labor share is constant at one-third of all expenditures in all sectors except Chemicals. Capital accounts for the largest expenditure share in all industries at about one-half, again except chemicals. Material shares are relatively small and vary substantially across sectors. Energy shares are the smallest, according to Table 1, but these data may not be sufficient to reflect an accurate picture because of the structural changes brought about by two oil price shocks during this period.

The second part of Table 1 shows the expenditure shares of the metals. Aluminum is most important in the Fabricated Metals, Cans and Containers sector. Copper dominates the Electricity sector. Nickel, tin and lead shares are significant only in the sector of Chemicals. Steel has been the most important metal in all industrial sectors except for Chemicals. Zinc has the smallest weights in all sectors.

If we compare the metal expenditure shares in 1964 and 1983, the steel share has declined sharply in the Fabricated Metals, Cans and Containers sector (from 75% to 50%), while it has remained unchanged in the Transportation sector and has increased in the Machinery and Electricity sectors. The aluminum share has increased in all industries but most sharply in Fabricated Metals, Cans and Containers sector (from 10% to 45%). Copper has declined in all sectors while nickel has increased in all sectors.

Table 1: Expenditure Shares For Selected U.S. Industrial Sectors (1964-1983 Average, in %)

	Chemicals	Fabricated Metals	Machinery	Electricity	Transportation
Labor	14	35	32	31	31
Energy	9	3	3	2	2
Capital	77	53	55	57	49
Metals	0.4	9	10	10	18
Total	100	100	100	100	100

	Chemicals	Fabricated Metals	Machinery	Electricity	Transportation
Aluminum	0	24	4	12	8
Copper	0	3	5	38	3
Nickel	55	3	1	4	2
Steel	0	64	88	41	81
Tin	19	5	1	2	1
Zinc	0	0	1	2	2
Lead	26	1	0	1	3
Total	100	100	100	100	100

Source: IECCM, The World Bank

Tables 2 presents the price elasticities of demand for energy, capital, labor and metals at mean prices in five U.S. industries.[14] An entry e_{ij} in these four by four matrices gives the percentage change in demand of the i^{th} input to the percentage change in price of the j^{th} input.

[14]Estimates of coefficients in the functional forms and associated standard errors are not reported in this paper.

Table 2: Estimated Demand Elasticities For Labor, Capital, Energy And Metals in Five U.S. Industrial Sectors (1964-1983)

Chemicals

	Labor	Capital	Energy	Metals
Labor	-0.185	0.078	0.109	-0.002
Capital	0.014	-0.041	0.028	-0.0005
Energy	0.149	0.208	-0.363	0.007
Metals	-0.027	-0.039	0.068	-0.001

Fabricated Metals, Cans & Containers

	Labor	Capital	Energy	Metals
Labor	-0.235	0.171	0.052	0.011
Capital	0.112	-0.276	0.017	0.146
Energy	0.565	0.281	-0.275	-0.570
Metals	0.028	0.553	-0.130	-0.451

Machinery

	Labor	Capital	Energy	Metals
Labor	-0.422	0.054	0.086	0.282
Capital	0.030	-0.050	0.022	-0.0008
Energy	1.238	0.574	-0.688	-1.124
Metals	0.664	-0.003	-0.184	-0.477

Table 2 (contd)

Electrical Equipment

	Labor	Capital	Energy	Metals
Labor	-0.476	0.355	0.066	0.054
Capital	0.182	-0.199	-0.0004	0.017
Energy	0.938	-0.011	-0.405	-0.522
Metals	0.157	0.098	-0.107	-0.148

Transportation Equipment

	Labor	Capital	Energy	Metals
Labor	-0.601	0.215	0.023	0.363
Capital	0.136	-0.223	0.012	0.075
Energy	0.341	0.265	-0.275	-0.331
Metals	0.612	0.198	-0.038	-0.771

Source: IECCM, The World Bank

The demand elasticities calculated with the SGM functional form can be compared with the results from the translog functional estimated by Priovolos and Dunietz.[15] In most cases, the two functional forms give similar results. Labor is substitutable with capital and energy in all five industries. Labor and metals are substitutes in all industries except Chemicals. However, capital and energy are found to be substitutes in all industries except in the Electrical Equipment sector, which is basically the opposite of what Priovolos and Dunietz (1987) found.[16] Capital and metals are substitutes in the Fabricated Metals, Electrical Equipment and Transportation Equipment sectors and complements in Chemicals

[15]See Appendix I.

[16]The study by Najmabadi and Imran (1987) suggests that capital and energy are complements in the short run and substitutes in the long run.

and Machinery. Energy and metals are complements except in Chemicals. The cross elasticities of energy demand with respect to material prices are significantly larger than the metals demand response with respect to energy prices.

The elasticities in Table 2 are measured at the means of the corresponding variables, and there is no guarantee that the elasticities in each year do not deviate from the elasticities at their means. Appendix II shows the graphs of the own-price elasticities for energy, capital, labor and metals for the five industries. One common feature is that the own-price elasticities for energy in all five industrial sectors begin to increase in absolute terms after 1973. Estimates start in the neighborhood of -0.10 in 1964 and end up in the range -1.0 to -4.0 by the early 1980s. The biggest jump comes in the period 1979 to 1981.

One of the reasons for this change in the estimated energy elasticity was probably a slow adjustment to the increase in oil prices in 1973. In the short run, due to fixed production procedures, industries had difficulties reducing energy consumption in the few years immediately after 1973. Thus the reduction in energy consumption was seen later at the time of the second sharp increase in oil prices. It is also likely that the once-and-for-all jump in oil prices in 1973 caused industries to be more responsive to changes in prices. To support this, Appendix II results show that all four curves of the own-price elasticities of energy are monotonically increasing after 1973.

Cross elasticities of demand for metals at the means of the sample period are presented in Table 3. Both the direct elasticities and the overall elasticities are presented.[17] The direct elasticities refer to the demand elasticity of the metal with respect to the price of the metal while holding aggregate metal consumption constant. The overall elasticity is the demand elasticity of the metal with respect to the metal price while allowing aggregate metal consumption to vary.

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[17]For definition of the direct and the overall cross elasticities, see equation (18).

Table 3: **Demand Elasticities For Metals In Five U.S. Industrial Sectors (1964-1983)**

CHEMICALS

OVERALL ELASTICITIES

	NICKEL	TIN	LEAD
NICKEL	-0.00030695	-0.0001530	-0.00010321
TIN	-0.00045191	-0.0022843	0.00089434
LEAD	-0.00023951	0.0007027	-0.00053495

DIRECT ELASTICITIES

	NICKEL	TIN	LEAD
NICKEL	-0.00001201	-0.0000531	0.00002389
TIN	-0.00015697	-0.0021845	0.00102144
LEAD	0.00005544	0.0008026	-0.00040785

FABRICATED METALS, CANS & CONTAINERS

OVERALL ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	LEAD
ALUMINUM	-0.17364	-0.057933	-0.014593	-0.17993	-0.021120	-0.007657
COPPER	-0.16081	-0.066023	-0.007575	-0.18825	-0.025137	-0.007079
NICKEL	-0.17478	-0.032685	-0.041464	-0.18375	-0.014075	-0.008120
STEEL	-0.15985	-0.060250	-0.013630	-0.19001	-0.024241	-0.006889
TIN	-0.15103	-0.064759	-0.008404	-0.19512	-0.032020	-0.003539
LEAD	-0.17673	-0.058861	-0.015647	-0.17897	-0.011423	-0.013249

DIRECT ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	LEAD
ALUMINUM	-0.008381	0.0016023	-0.000795	0.0060816	0.0019895	-0.0004970
COPPER	0.004448	-0.0064885	0.006223	-0.0022355	-0.0020277	0.0000811
NICKEL	-0.009523	0.0268499	-0.027666	0.0022641	0.0090352	-0.0009595
STEEL	0.005403	-0.0007155	0.000168	-0.0039951	-0.0011316	0.0002713
TIN	0.014227	-0.0052237	0.005395	-0.0091089	-0.0089101	0.0036210
LEAD	-0.011472	0.0006743	-0.001849	0.0070491	0.0116871	-0.0060892

Table 3 (contd)

MACHINERY

OVERALL ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC
ALUMINUM	-0.021407	-0.017410	-0.01094	-0.41066	-0.00787	-0.00898
COPPER	-0.014071	-0.060060	0.02114	-0.46293	0.01850	0.02015
NICKEL	-0.044698	0.106917	-0.19575	-0.24248	-0.03333	-0.06793
STEEL	-0.019658	-0.027418	-0.00284	-0.42317	-0.00238	-0.00180
TIN	-0.042472	0.123458	-0.04398	-0.26778	-0.18190	-0.06459
ZINC	-0.049222	0.136642	-0.09111	-0.20653	-0.06564	-0.20141

DIRECT ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC
ALUMINUM	-0.001303	0.007465	-0.00602	0.009315	-0.00415	-0.00531
COPPER	0.006033	-0.035185	0.02606	-0.042952	0.02223	0.02381
NICKEL	-0.024594	0.131791	-0.19083	0.177494	-0.02960	-0.06426
STEEL	0.000446	-0.002544	0.00208	-0.003196	0.00135	0.00186
TIN	-0.022368	0.148333	-0.03906	0.152191	-0.17817	-0.06092
ZINC	-0.029118	0.161516	-0.08619	0.213443	-0.06191	-0.19774

Table 3 (contd)

ELECTRICAL EQUIPMENT

OVERALL ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	-0.27456	-0.052173	0.03452	0.11369	0.05633	-0.00989	-0.01619
COPPER	-0.01651	-0.057151	-0.00585	-0.06220	-0.00295	-0.00261	-0.00101
NICKEL	0.10868	-0.058225	-0.13296	-0.21778	0.04369	0.10347	0.00485
STEEL	0.03407	-0.058918	-0.02073	-0.10866	-0.00729	0.00832	0.00494
TIN	0.35383	-0.058656	0.08718	-0.15289	-0.21631	-0.16094	-0.00049
ZINC	-0.06168	-0.051376	0.20499	0.17315	-0.15980	-0.24096	-0.01260
LEAD	-0.18153	-0.035792	0.01728	0.18491	-0.00088	-0.02265	-0.10961

DIRECT ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	-0.25653	0.004816	0.04025	0.17385	0.05920	-0.00700	-0.01458
COPPER	0.00152	-0.000161	-0.00012	-0.00204	-0.00008	0.00028	0.00060
NICKEL	0.12671	-0.001235	-0.12724	-0.15762	0.04656	0.10636	0.00646
STEEL	0.05210	-0.001929	-0.01500	-0.04850	-0.00442	0.01121	0.00655
TIN	0.37186	-0.001666	0.09291	-0.09273	-0.21344	-0.15805	0.00112
ZINC	-0.04365	0.005613	0.21072	0.23331	-0.15693	-0.23807	-0.01099
LEAD	-0.16350	0.021197	0.02301	0.24507	0.00199	-0.01976	-0.10801

Table 3 (contd)

TRANSPORTATION EQUIPMENT

OVERALL ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	-0.06916	-0.03529	0.01113	-0.65333	0.00027	-0.01857	-0.029666
COPPER	-0.11682	-0.19824	0.30706	-0.65953	0.00926	-0.06418	-0.072162
NICKEL	0.04663	0.38876	-0.80174	-0.67612	0.03860	0.14561	0.063656
STEEL	-0.06163	-0.01879	-0.01522	-0.66209	-0.00285	-0.00988	-0.024157
TIN	0.00371	0.03794	0.12485	-0.40952	-0.40075	-0.17419	0.023346
ZINC	-0.09422	-0.09838	0.17629	-0.53132	-0.06520	-0.22587	0.044085
LEAD	-0.07612	-0.05593	0.03897	-0.65708	0.00442	0.02229	-0.071168

DIRECT ELASTICITIES

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	-0.007091	-0.01654	0.02594	0.004656	0.00485	-0.00634	-0.005476
COPPER	-0.054751	-0.17949	0.32187	-0.001552	0.01384	-0.05195	-0.047971
NICKEL	0.108694	0.40751	-0.78693	-0.018134	0.04317	0.15784	0.087846
STEEL	0.000439	-0.00004	-0.00041	-0.004104	0.00173	0.00235	0.000033
TIN	0.065774	0.05669	0.13966	0.248466	-0.39617	-0.16196	0.047536
ZINC	-0.032152	-0.07963	0.19110	0.126662	-0.06062	-0.21364	0.068275
LEAD	-0.014050	-0.03718	0.05378	0.000907	0.00900	0.03453	-0.046978

Source: IECCM, The World Bank

The estimates of the price elasticities for three major metals (nickel, tin, lead) used in the Chemicals sector, have the same signs as Priovolos and Dunietz(1987),[18] with the exception that the own-price elasticity of nickel is negative in the SGM model. Metals comprise only 0.3% of the total factor costs in the Chemicals sector. Because of the technical nature of the industry, all the own and substitution elasticities are very inelastic.

In the results obtained by Priovolos and Dunietz(1987), the own elasticities for aluminum, copper and tin were found to be positive in the Fabricated Metals, Cans & Containers sector. There are no cases of non-negative own elasticities from the SGM model. In the overall elasticities, aluminum was found to be complementary to all other

[18]See Appendix III.

metals. Pairwise, steel - aluminum and copper - aluminum do not show the expected substitutability. However, steel - aluminum and copper - aluminum are found to be substitutes at the disaggregated level. These results can be explained as follows: we observe complementarity between steel - aluminum and between copper - aluminum in terms of overall cross elasticities based on the combination of two effects. First, if the price of aluminum increases and prices of all other metals are held constant, steel demand or copper demand will increase at the disaggregated level because of substitutability. Second, as the aluminum price increases, the overall input demand for metals at the aggregate level declines, thus reducing the steel and copper inputs. The net effect of these two opposite moves determines the overall input demand for steel and copper.

In the Machinery sector, the results are similar to those obtained by Priovolos and Dunietz(1987). Aluminum is complementary to all other metals based on the overall elasticities, but a substitute to steel and copper based on the direct elasticities. Durability and cost are the primary characteristics of the goods produced in this sector. Carbon steel is the preferred material in most uses and has no major competitor in the market. Steel has the largest overall own-price elasticity, but its direct own-price elasticity is the second smallest. These results reflect the fact that steel accounts for over 80% of total metal expenditures in the sector. A change in the steel price will change direct steel demand very little if total metal consumption is constant. However, a change in the steel price will change the total metal price index significantly, so that the demand for total metal will adjust as well as the demand for steel. The dominance of steel is also supported by the sign of the cross elasticities of demand with other metals. It is found that steel is complementary to all other metals in terms of the overall elasticities. A steel price increase will bring down consumption of other metals.

Compared to the findings of Priovolos and Dunietz, in the Electrical Equipment sector, SGM results do not show aluminum - copper to be substitutes, nor are aluminum - steel complements in terms of the overall cross elasticities. However, the direct cross elasticities show that aluminum - copper and aluminum - steel are substitutes as in the other two industrial sectors discussed above. Because total metal consumption in the Electrical equipment sector only accounts for around 10% of the total cost and the own-price elasticity of total metals is small compared to the situation in the other two industrial sectors, the adjustment from direct elasticities to overall elasticities is also small.

Steel dominates metal consumption in the Transportation Equipment sector. On average over the sample period, steel costs made up more than 80% of total metal expenditures. Similar to the findings of Priovolos and Dunietz(1987), strong complementarity exists between all other metals and steel in terms of the overall price elasticities. Consistent with all other industrial sectors, steel and aluminum are substitutes in terms of direct elasticities, but the absolute value of their cross elasticity is virtually zero. This can be explained by the preference of U.S. consumers towards bigger cars and the slow adjustment of U.S. auto makers towards smaller cars in the years after the first oil shock. Also, although the average automobile now weighs much less than in earlier years, aluminum is not the only substitute for carbon steel. High-strength steel, plastics, composite metals and galvanized and other coated steel sheets also played an important role. Surprisingly, aluminum and copper are always complements, either in the overall or direct elasticities. This result arises perhaps because the copper usage is only one-fifth that of aluminum, and therefore, substitution of aluminum for copper has never played a significant role in the form of the technological advancement.

VI. Bootstrapping Selected Elasticity Estimates

As discussed earlier, the bootstrap procedure can be used to estimate standard errors for complicated systems such as SGM. This procedure can be used to deal with non-normally distributed errors, lag structures, and simultaneous nonlinear maximum likelihood estimators.[19] The technique proceeds from the point where the model has been fitted to data by some statistical procedure and there are residuals, namely the difference between the observed and fitted values. The key idea is to resample the residuals, preserving the known or unknown distribution properties. Assuming the model and the estimated parameters to be correct, the resampling generates 'pseudo-data'. The model then can be refitted.

Our sample period is from 1964 to 1983, 20 observations in all. Denote $u_t=(u_{t1},...,u_{tN})^T$ to be the vector of error terms of the SGM estimation, with $E(u_t)=0$ and $E(u_t u_t^T)=\Omega$. In each Bootstrap trial a random generator creates a series of univariate distributed repeatable integers between one and twenty. This series of integers is used to scramble the original error structure of the system $(u_1,...,u_{20})$ to be $(u_{s1},...,u_{s20})$, where subscript $si=j$ and $i,j=1,2,...,20$. The estimation process is applied backwards, using the estimated parameters to create a set of pseudo-data for endogenous variables in the system. The model is then re-estimated. Fifty Bootstrap trials were completed.[20]

Table 4 demonstrates the results from the Bootstrap trials for the Chemicals and Fabricated Metals, Cans & Containers sectors. Both means and standard deviations from the simulations are presented.

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[19]The SGM forms estimated in this paper are assumed to have a multivariate normal distributed error structure. Regular Monte Carlo simulation is also appropriate.

[20]Because of the great amount of programming and calculation involved, bootstrapping were done only for the cross elasticities in the Chemicals industry at the aggregate level, and in the Fabricated Metals, Cans & Containers industry at the detailed level.

Table 4: **Bootstrap Results On Cross Elasticities (1964-1983)**

CHEMICALS (AGGREGATE LEVEL)

MEANS

	LABOR	CAPITAL	ENERGY	METAL
LABOR	-0.18538	0.078484	0.10902	-0.0021215
CAPITAL	0.01422	-0.041270	0.02760	-0.0005497
ENERGY	0.14859	0.207561	-0.36332	0.0071672
METAL	-0.02759	-0.039452	0.06839	-0.0013501

STANDARD DEVIATIONS

	LABOR	CAPITAL	ENERGY	METAL
LABOR	0.00195289	0.00075284	0.00123417	0.0000223059
CAPITAL	0.00014156	0.00047074	0.00033440	0.0000071660
ENERGY	0.00153360	0.00222377	0.00370468	0.0000950558
METAL	0.00072594	0.00125298	0.00204086	0.0000644755

Table 4 (contd)

FABRICATED METALS, CANS & CONTAINERS (METALS)

OVERALL ELASTICITIES

MEANS

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	LEAD
ALUMINUM	-0.17057	-0.057888	-0.013540	-0.18302	-0.022060	-0.007099
COPPER	-0.16229	-0.067935	-0.005687	-0.18892	-0.023613	-0.005737
NICKEL	-0.16276	-0.032656	-0.049988	-0.17981	-0.021178	-0.007784
STEEL	-0.16013	-0.058836	-0.012734	-0.19327	-0.022048	-0.007163
TIN	-0.15867	-0.060647	-0.012076	-0.18116	-0.040186	-0.001435
LEAD	-0.16515	-0.047196	-0.014559	-0.19035	-0.004622	-0.032305

STANDARD DEVIATIONS

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	LEAD
ALUMINUM	0.0225294	0.0090765	0.0042799	0.0250600	0.0030897	0.0021263
COPPER	0.0245221	0.0104503	0.0125661	0.0285112	0.0042758	0.0048054
NICKEL	0.0478849	0.0381409	0.0563171	0.0481748	0.0187244	0.0148864
STEEL	0.0210413	0.0085786	0.0038111	0.0242343	0.0026642	0.0017295
TIN	0.0269702	0.0130536	0.0110181	0.0296763	0.0135140	0.0080126
LEAD	0.0517322	0.0398801	0.0287030	0.0487864	0.0261031	0.0317292

Table 4 (contd)

DIRECT ELASTICITIES

MEANS

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	LEAD
ALUMINUM	-0.0061831	0.0007029	-0.000053	0.0047754	0.000798	-0.000040
COPPER	0.0020933	-0.0093443	0.007800	-0.0011160	-0.000755	0.001322
NICKEL	0.0016266	0.0259340	-0.036501	0.0079862	0.001680	-0.000726
STEEL	0.0042592	-0.0002451	0.000753	-0.0054726	0.000810	-0.000105
TIN	0.0057128	-0.0020561	0.001410	0.0066367	-0.017328	0.005624
LEAD	-0.0007611	0.0113943	-0.001072	-0.0025501	0.018236	-0.025247

STANDARD DEVIATIONS

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	LEAD
ALUMINUM	0.0080245	0.0039724	0.0033144	0.0063463	0.0025409	0.0020876
COPPER	0.0112269	0.0107233	0.0105595	0.0102677	0.0039085	0.0048505
NICKEL	0.0401117	0.0413358	0.0548462	0.0355716	0.0186352	0.0149091
STEEL	0.0056069	0.0030351	0.0023962	0.0048627	0.0022709	0.0016404
TIN	0.0180092	0.0099114	0.0109382	0.0185208	0.0132448	0.0080298
LEAD	0.0481719	0.0395009	0.0281793	0.0434032	0.0261964	0.0317207

Source: IECCM, The World Bank

From the simulation results on the aggregate level in the Chemical sector it is easy to see that the means of the simulation trials for the cross price elasticities are almost identical to their actual estimates in Table 1. This result confirms that the univariate random generator used in the Bootstrap procedure is appropriate in terms of its unbiasedness. The standard deviations for cross elasticity estimates in Chemicals are significantly smaller than the estimates themselves, which indicates that the elasticity estimates are fairly stable, and statistically different from zero.

Table 4 also presents bootstrap simulation results for the overall and direct cross elasticities at the individual metal level for the Fabricated Metals, Cans and Containers sector. In the direct elasticity case, the means of 50 trials are slightly different from their original estimations, and the standard deviations are relatively larger, which implies that some of the estimates are not significantly different from zero. However, the absolute magnitude of either the original estimates or their standard deviations are fairly small, indicating that zero cross elasticities are perhaps the stable solution in the SGM system.

The means of the overall elasticities from the simulations are not different from their estimates in Table 3. About two-thirds of the overall elasticities in this sector appear to be statistically significant. It is easy to see that because of the separability assumption used in the SGM system, the randomness introduced in the Bootstrap trials on the detailed level reaches the aggregate level only through changes in the relative shares of a particular metal to the total material group.[21] Thus, the stability of the overall elasticities of metals comes primarily from the stability of the estimates of the own-price elasticity of metals at the aggregate level.

Finally, Appendix IV presents, the Bootstrapping results in the form of confidence intervals[22] on the own elasticities in the Chemical sector. Recall that the most notable result in Appendix II is the significant jump in energy own-price elasticities in all five industrial sectors in the late 1970s. Standard error calculations would help identify the rationale behind those results. As Appendix IV shows, the means of the simulation through time are almost identical to the actual estimations. Their corresponding standard errors are significantly smaller. This suggests that the change in the own elasticity of demand could be structural. On the other hand, the heteroscedasticity does deserve some attention as the standard error gradually rises following the time trend.

In summary, according to the results from bootstrapping simulations, it appears that the SGM is capable of dealing with a system which includes four inputs within a sample size of 20 observations, i.e., at the aggregate level of the Chemicals industry. In this case the solutions of the system likelihood function are insensitive to the data resampling. However, when the number of inputs increases to six, as in the case of Fabricated Metals, Cans & Containers sector at the detailed metal level, the solutions of the likelihood function are more sensitive to the random shocks on residuals, and the estimations become insignificant.

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[21]The whole parameter structure at the aggregate level is not touched.

[22]Add or subtract two times the standard error from the simulated mean.

VII. Conclusions

The SGM functional form yields results consistent with the translog functional form in most cases. It has the advantage of imposing the concavity constraint globally on the cost function and of ensuring that the results satisfy the economic theory. Moreover, the procedure is able to distinguish between the direct elasticities and the overall elasticities under the separability assumption adopted for the metal subgroup.

However, the procedure has some disadvantages. The estimation process is complicated; it requires many free parameters, which makes small-sample estimation impractical. Like all other flexible functional forms, SGM does not incorporate a dynamic structure and therefore the elasticities estimated are short-term elasticities only. More seriously, if in reality the firm's behavior does not fit the assumed static, one-period optimization, the regularity conditions in the one-period cost function do not necessarily hold, and the enforcement of the regularity conditions by SGM may be somewhat misleading.

In terms of empirical results, we do see evidence of structural changes in U.S. industry, i.e., the jump in the own-price elasticities of energy in all five industries covered in the paper. Although the SGM flexible functional form agrees with the translog results on the complementarity of aluminum and steel, except in the Electrical Equipment sector, it does suggest that aluminum and steel are substitutes in the technical-compensated sense; that is, when total metals use is held constant, an increase in the price of one metal does reduce the consumption of that metal and increases the consumption of others.

Bootstrapping techniques provide insights into the stability of the elasticity estimation, and the results are promising at the aggregate level when the number of free parameters is not large compared to the sample size. Bootstrapping also makes more clear the problem on the disaggregated level of estimation where most of the elasticities are insignificantly different from zero.

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Appendix I

DEMAND ELASTICITIES FOR LABOR, CAPITAL, ENERGY AND METALS IN FIVE US INDUSTRIAL SECTORS

CHEMICALS	LABOR	CAPITAL	ENERGY	METALS
LABOR	-0.131	0.149	0.009	-0.028
CAPITAL	0.028	-0.077	0.040	0.009
ENERGY	0.016	0.357	-0.348	-0.026
METALS	-0.908	1.580	-0.495	-0.179

FABRICATED METALS, CANS & CONT.	LABOR	CAPITAL	ENERGY	METALS
LABOR	-0.402	0.305	0.032	0.065
CAPITAL	0.203	-0.277	-0.007	0.081
ENERGY	0.421	-0.129	-0.378	0.086
METALS	0.249	0.465	0.025	-0.739

MACHINERY	LABOR	CAPITAL	ENERGY	METALS
LABOR	-0.460	0.354	0.036	0.070
CAPITAL	0.202	-0.333	-0.005	0.135
ENERGY	0.637	-0.141	-0.537	0.041
METALS	0.167	0.565	0.005	-0.737

	ELECTRICAL EQUIPMENT			
	LABOR	CAPITAL	ENERGY	METALS
LABOR	-0.576	0.461	0.021	0.094
CAPITAL	0.249	-0.267	0.002	0.016
ENERGY	0.356	0.061	-0.377	-0.040
METALS	0.271	0.084	-0.007	-0.348

	TRANSPORTATION EQUIPMENT			
	LABOR	CAPITAL	ENERGY	METALS
LABOR	-0.761	0.403	0.023	0.335
CAPITAL	0.259	-0.243	-0.005	-0.011
ENERGY	0.403	-0.138	-0.202	-0.064
METALS	0.570	-0.029	-0.006	-0.535

SOURCE: THE WORLD BANK, INTERNATIONAL ECONOMICS DEPARTMENT.

Appendix II

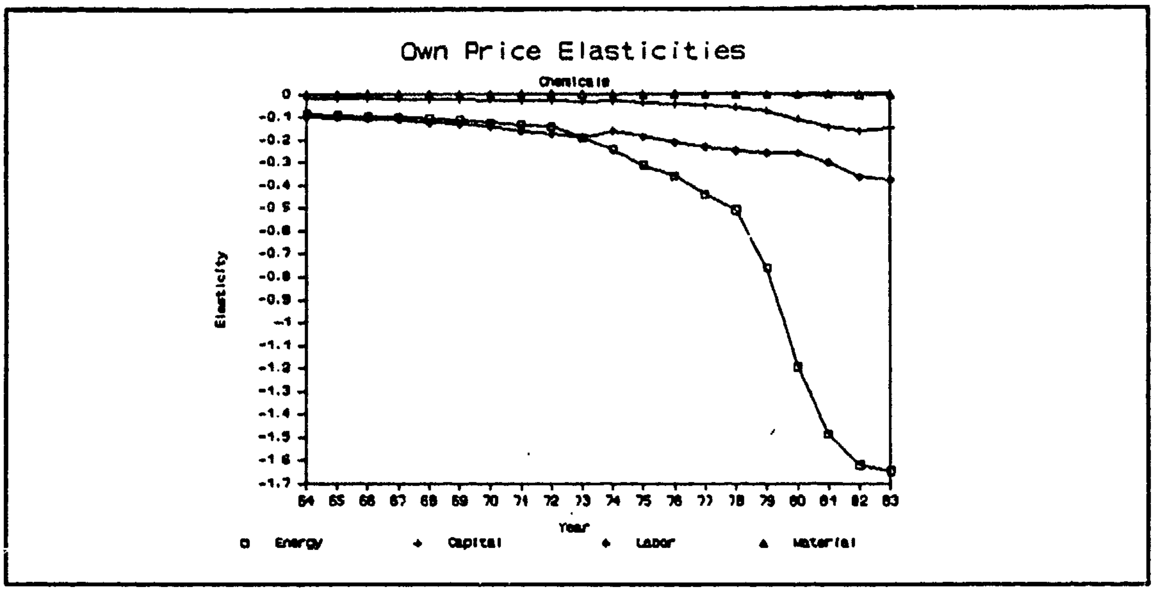


Figure 1
Source: IECCM, The World Bank

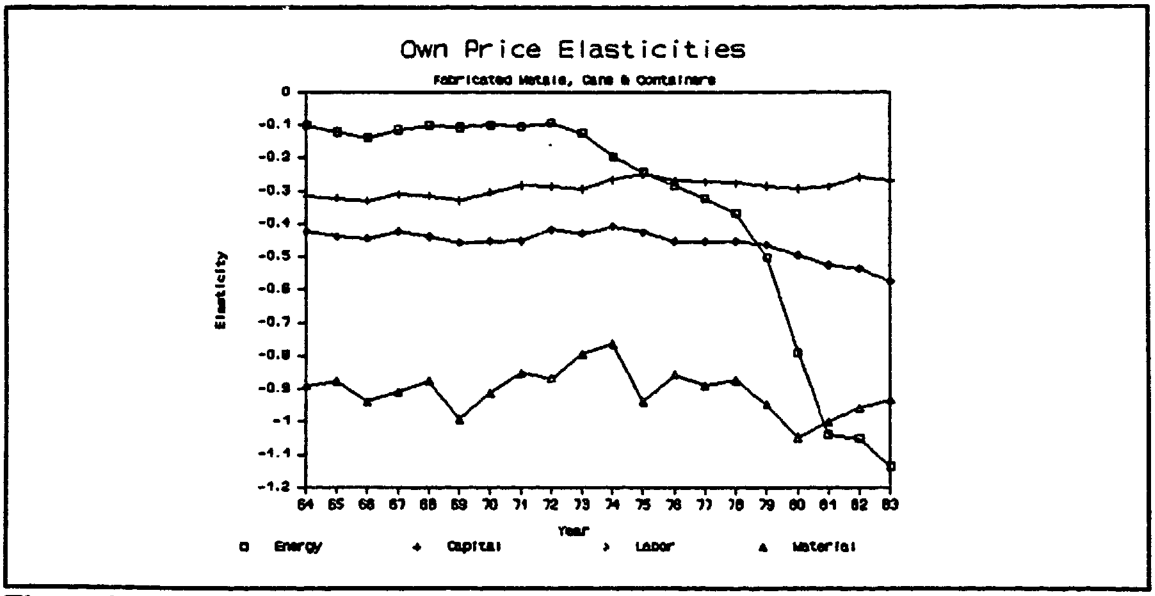


Figure 2
Source: IECCM, The World Bank

Appendix II

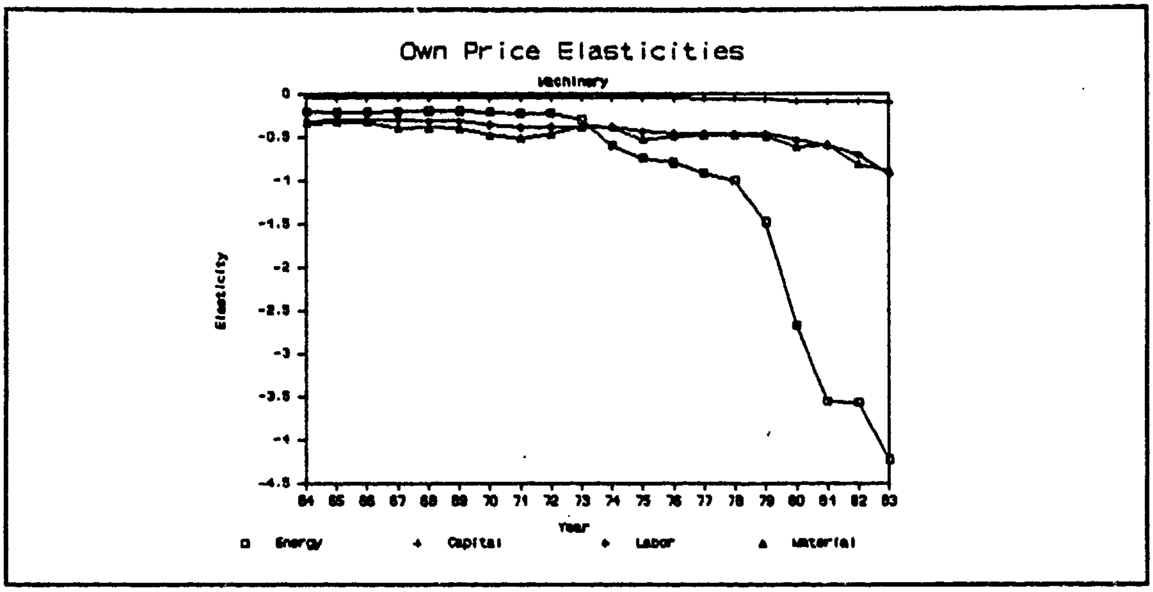


Figure 3
Source: IECCM, The World Bank

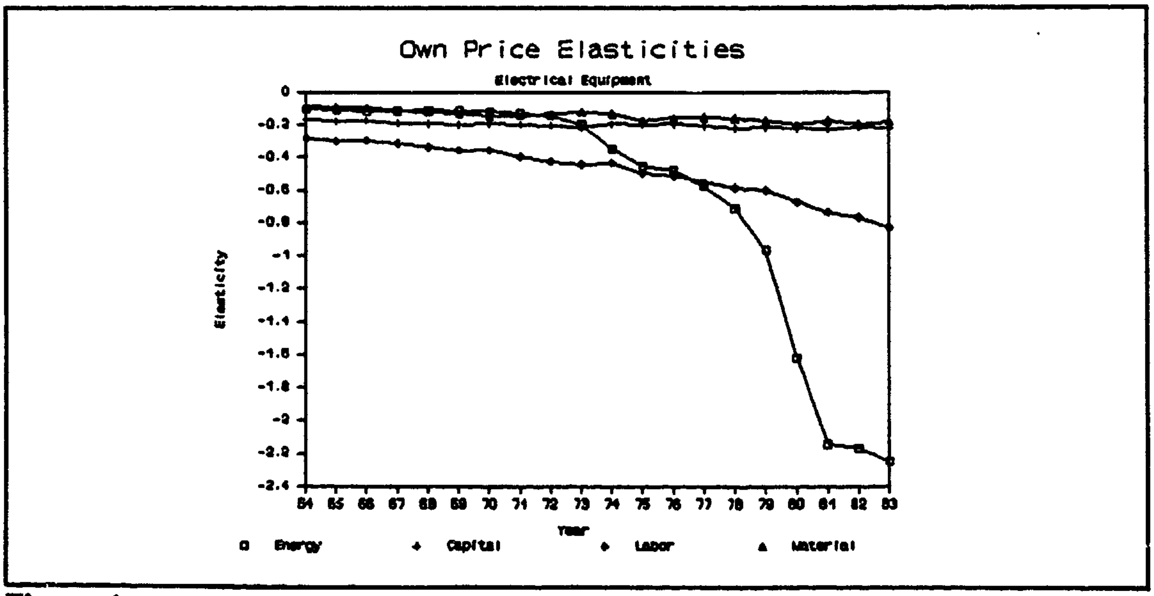


Figure 4
Source: IECCM, The World Bank

Appendix II

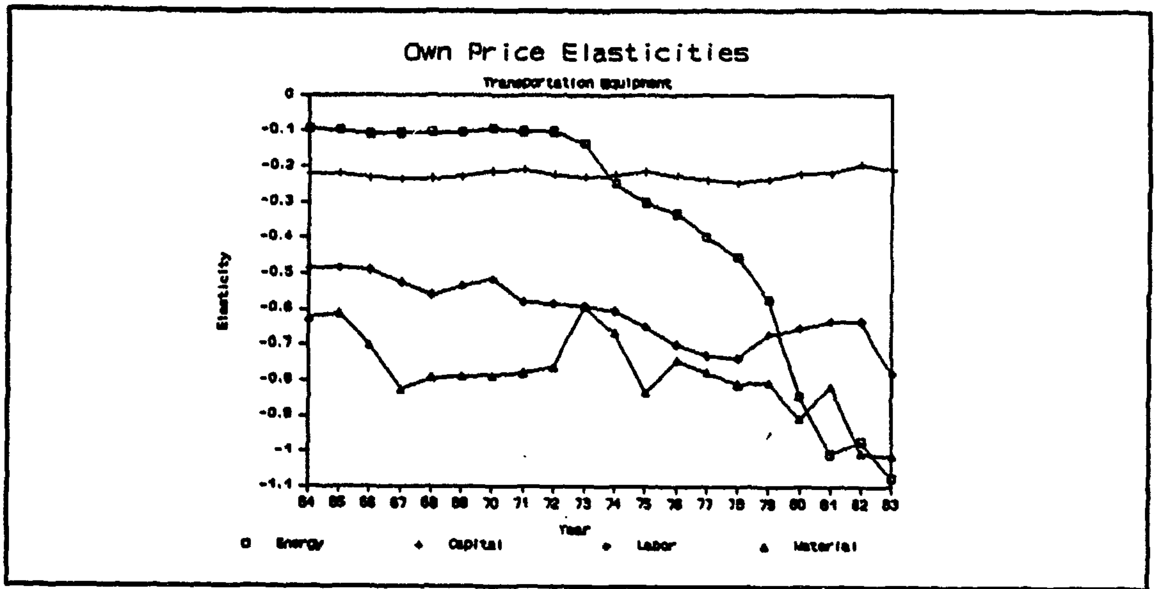


Figure 5
Source: IECCM, The World Bank

Appendix III

DEMAND ELASTICITIES FOR ALUMINUM, COPPER, NICKEL, STEEL, TIN, ZINC AND LEAD IN FIVE US INDUSTRIES

CHEMICALS

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	0.000	0.000	0.000	0.000	0.000	0.000	0.000
COPPER	0.000	0.000	0.000	0.000	0.000	0.000	0.000
NICKEL	0.000	0.000	0.243	0.000	-0.014	0.000	-0.282
STEEL	0.000	0.000	0.000	0.000	0.000	0.000	0.000
TIN	0.000	0.000	-0.392	0.000	-0.350	0.000	0.563
ZINC	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LEAD	0.000	0.000	-0.589	0.000	0.428	0.000	-0.018

FABRICATED METALS, CANS & CONT.

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	0.099	-0.029	-0.206	-0.505	-0.060	0.000	-0.038
COPPER	-0.202	0.186	-0.342	0.014	-0.378	0.000	-0.016
NICKEL	-1.867	-0.447	-0.170	1.535	0.171	0.000	0.040
STEEL	-0.190	0.001	0.064	-0.595	-0.028	0.000	0.011
TIN	-0.267	-0.244	0.084	-0.340	0.092	0.000	-0.064
ZINC	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LEAD	-0.334	-0.050	0.095	0.620	-0.312	0.000	-0.258

MACHINERY

	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	-0.377	0.021	-0.001	-0.297	0.003	-0.087	0.000
COPPER	0.016	-0.081	-0.212	-0.334	-0.026	-0.101	0.000
NICKEL	-0.006	-1.109	-0.146	-0.217	0.291	0.448	0.000
STEEL	-0.014	-0.020	-0.002	-0.705	-0.006	0.010	0.000
TIN	0.019	-0.183	0.396	-0.713	-0.178	-0.079	0.000
ZINC	-0.461	-0.684	0.582	1.165	-0.075	-1.264	0.000
LEAD	0.000	0.000	0.000	0.000	0.000	0.000	0.000

	ELECTRICAL EQUIPMENT						
	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	-0.356	0.436	-0.154	-0.272	0.016	0.090	-0.108
COPPER	0.137	-0.204	-0.047	-0.191	-0.017	-0.039	0.012
NICKEL	-0.486	-0.466	0.170	0.028	0.244	0.276	-0.115
STEEL	-0.081	-0.175	0.003	-0.099	-0.009	-0.029	0.042
TIN	0.103	-0.355	0.504	-0.194	-0.056	-0.321	-0.030
ZINC	0.551	-0.744	0.540	-0.608	-0.303	0.357	-0.142
LEAD	-1.213	0.426	-0.409	1.562	-0.052	-0.259	-0.404

	TRANSPORTATION EQUIPMENT						
	ALUMINUM	COPPER	NICKEL	STEEL	TIN	ZINC	LEAD
ALUMINUM	-0.280	0.005	0.010	-0.148	-0.007	-0.035	-0.080
COPPER	0.015	0.049	-0.317	-0.009	0.014	-0.304	0.018
NICKEL	0.043	-0.414	-0.289	-0.218	0.029	0.385	-0.071
STEEL	-0.015	0.000	-0.005	-0.514	-0.001	-0.001	0.002
TIN	-0.097	0.060	0.098	-0.180	-0.170	-0.231	-0.015
ZINC	-0.174	-0.469	0.455	-0.039	-0.081	0.182	-0.407
LEAD	-0.210	0.015	-0.045	0.038	-0.003	-0.218	-0.110

SOURCE: THE WORLD BANK, INTERNATIONAL ECONOMICS DEPARTMENT.

Appendix IV

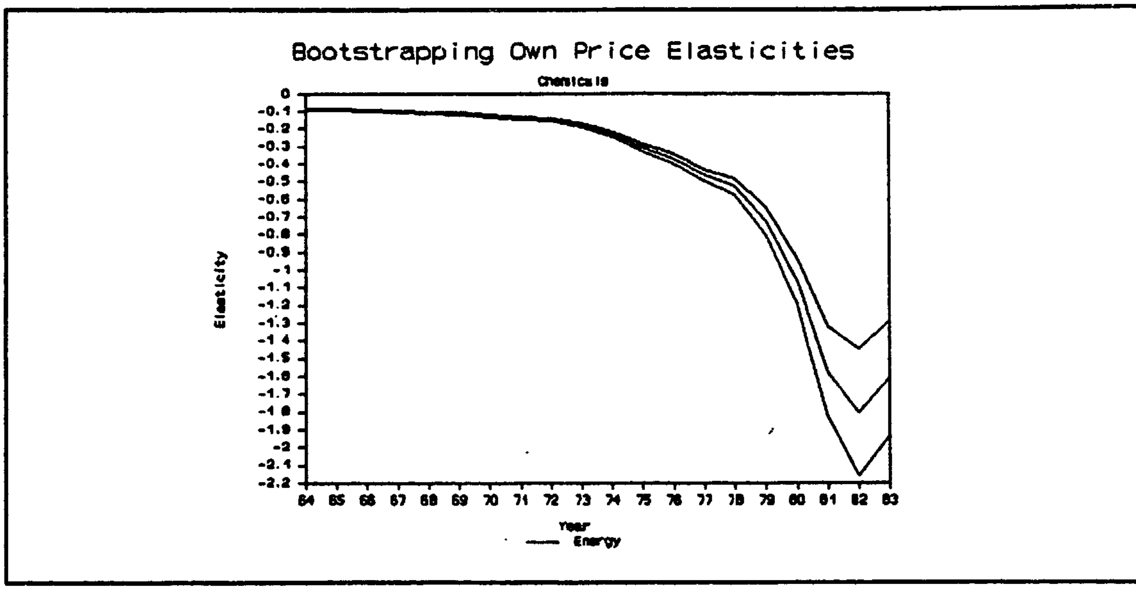


Figure 1
Source: IECCM, The World Bank

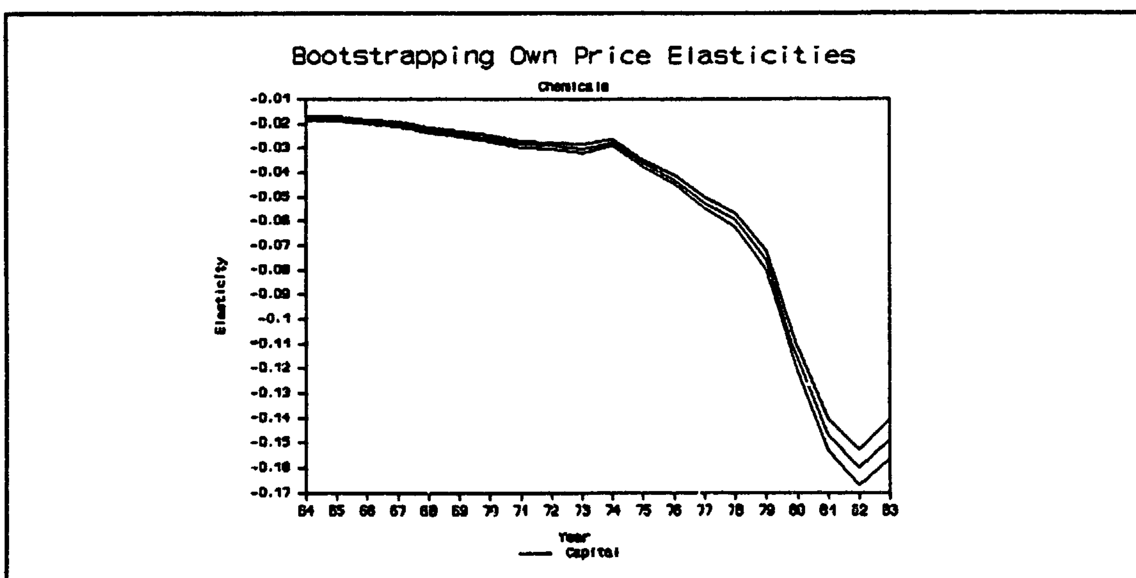
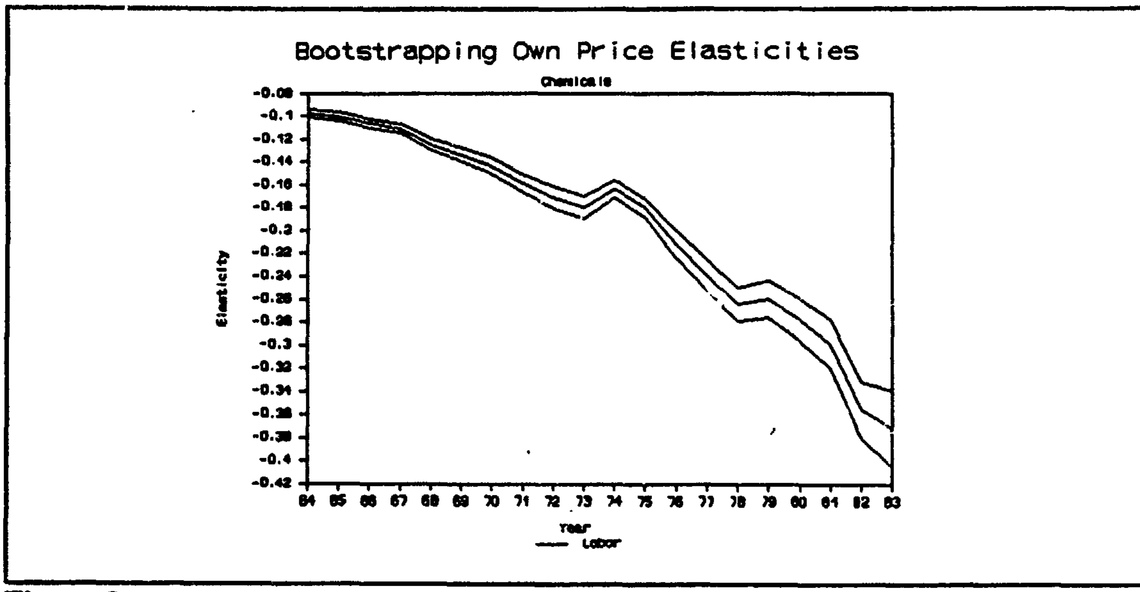


Figure 2
Source: IECCM, The World Bank

Appendix IV



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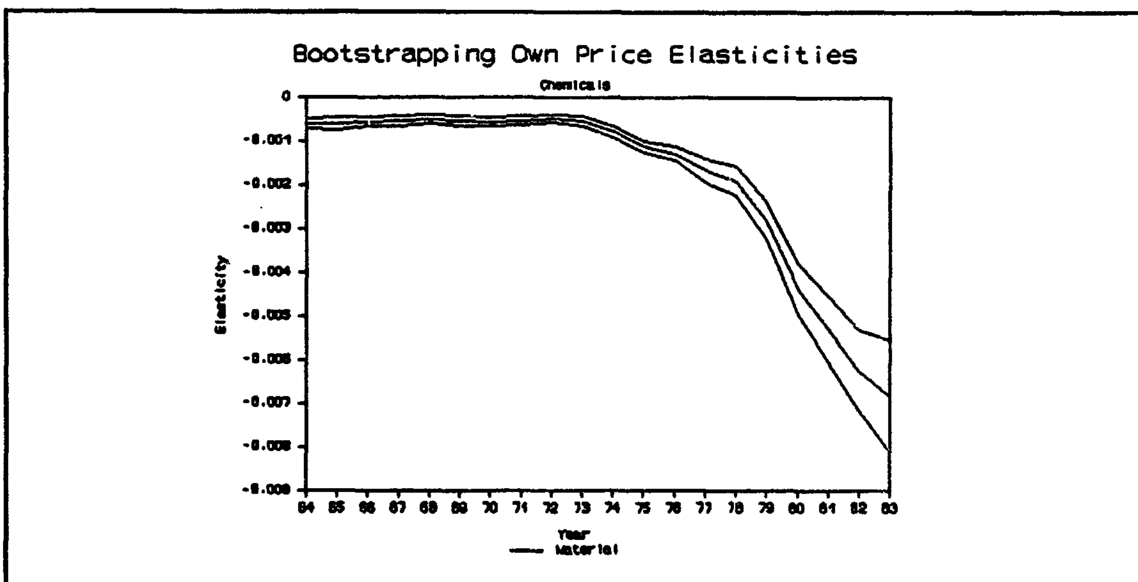

Figure 4
Source: IECCM, The World Bank

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